Universitat de Barcelona - Department of Economic Theory
Master in Economics

Macroeconomics I - Fall Semester 2011

Lorenzo Burlon

Problem Set 4.

Learning-by-doing and the role for public policy.

Consider the variant of the AK model that we saw in class. There are two types of capital, physical capital and human capital. Assume that there is no population growth and no technological progress. Normalize the size of the population to 1. Total output is given by

$$Y_t = F(K_t, H_t), \tag{1}$$

where K_t is aggregate physical capital at time t and H_t is aggregate human capital. The latter is defined as

$$H_t = h_t L_t, \tag{2}$$

where h_t is human capital per unit of labor (the *know-how*) and L_t is labor. The production function F is Neoclassical.

Unlike in the economy we analyzed in class, here both households and firms take h_t as given. Suppose that there exists a representative household for this economy and that we look at the decentralized allocations. The household takes the price path $\{w_t, r_t\}_{t=0}^{\infty}$ and the infinite stream of human capital per unit of labor $\{h_t\}_{t=0}^{\infty}$ as given, and maximizes utility

$$\mathscr{U}_0 = \sum_{t=0}^{+\infty} \beta^t U(c_t) \tag{3}$$

subject to the budget constraint

$$c_t + i_t + b_{t+1} \leq w_t l_t + r_t k_t + (1 + R_t) b_t. \tag{4}$$

Note that in this economy households do not decide how much human capital to accumulate, since they take h_t as given. We suppose that U is neoclassical.

Each firm m has access to the same production technology, that is,

$$Y_t^m = F(K_t^m, h_t L_t^m), \tag{5}$$

where K_t^m and L_t^m are firm-specific capital and labor. Hence, profits are

$$\Pi_t^m = F(K_t^m, h_t L_t^m) - r_t K_t^m - w_t L_t^m, \tag{6}$$

which firm m maximizes at time t taking the prices (w_t, r_t) and the know-how h_t as given. Once profits are realized, the firm distributes the profits in the form of dividends and disappears.

We assume that the know-how h_t stems from learning-by-doing, that is, it is the unintentional by-product of aggregate production. The higher the production per unit of labor (per hours worked) y_t, the higher the know-how. We suppose then that the level of know-how is proportional to the level of output per unit of labor, that is,

$$h_t = \nu y_t, \tag{7}$$

where $\nu > 0$ measures how effective the process of learning-by-doing is.

1. Use (1) and (7) in order to obtain that

$$y_t = a(\nu) k_t,$$

   where $a(\nu)$ is a real-valued function of ν. [Use the properties of neoclassical production functions. Remember that $f(x) \equiv F(x, 1)$ for any x and that its inverse function exists.]

2. Define the problem of the central planner and derive the Euler condition in the centralized economy. Derive the function $A^*(\nu)$ that makes the Euler condition look similar to the one in the AK model. [Use the simplest formulation of the central planner's problem.]

3. Define a general equilibrium in the decentralized economy. [Remember to include the process of learning-by-doing.]

4. Derive the FOCs for firm m. Can we talk about a representative firm in this economy? Express the FOCs in terms of $\kappa_t \equiv k_t/h_t$ and h_t. [Pay particular attention on w_t, it should depend on h_t.]

5. Derive the functions $A(\nu)$ and $\omega(\nu)$ such that in equilibrium $r_t = A(\nu)$ and $w_t = \omega(\nu)k_t$ for every t. What is the relation between $A^*(\nu)$, $A(\nu)$, and $\omega(\nu)$?

6. Derive the Euler condition of the household and substitute for the equilibrium value of r_t. What do you conclude about the relation between social return to capital and private return to capital? Would you say that the equilibrium allocation in this economy coincides with the Pareto-optimal allocation? [Do not list all the K-K-T conditions, derive simply the FOCs of the simplified problem of the household.]

7. Suppose we introduce a government that collects fiscal revenue through lump-sum taxation of the households and subsidizies the investment decisions of the household. In other words, the government balanced budget is

$$\sigma i_t = T_t,$$

where T_t are taxes and $\sigma > 0$ is the subsidy that the government commits to give to the representative household for each unit of income it devotes to investment. The budget constraint of the household becomes

$$c_t + i_t + b_{t+1} \leq w_t l_t + r_t k_t + (1 + R_t)b_t + \sigma i_t - T_t,$$

where σi_t is the whole subsidy that the government devotes to the household. What would be the new Euler condition of the household given the equilibrium level of r_t?

8. Can you find a level of subsidy σ such that the equilibrium allocation coincides with the Pareto-optimal allocation?

9. Analyze the dynamics implied by the equilibrium allocation with a low level of subsidies, $\sigma_L > 0$. Represent graphically these dynamics on the phase diagram (k_t, c_t).

10. Suppose that the economy starts with a low level of subsidies, σ_L. Suppose that at time $t = 0$, the government decides to increase permanently the subsidies from σ_L to σ_H. What are the short run and long run effects of such a policy on c_t and k_t?