Universitat de Barcelona - Department of Economic Theory
Master in Economics

Macroeconomics I - Fall Semester 2011

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Problem Set 3.

The Ramsey-Cass-Koopmans model with proportional income taxation and lump-sum transfers.

Consider the neoclassical growth model. Assume that there is no population growth and normalize the size of the population to 1. Suppose that there is no technological growth either. We introduce a government in the model. The government taxes the households and distributes the fiscal revenues through lump-sum transfers. The taxes are collected through proportional taxation on realized income, that is, a constant fraction $\tau \in (0, 1)$, the tax rate, of households' incomes goes into taxes. Note that the tax rate is constant over time. The total fiscal revenue will then be $\int_0^1 \tau y_t^j dj$, where y_t^j is the income of household j at time t. We suppose that the government runs a balanced budget policy, so that

$$\int_0^1 \tau y_t^j dj = T_t,$$

where $T_t \equiv \int_0^1 T_t^j dj$ is the total amount of lump-sum transfers distributed to the mass 1 of households.

Suppose that household j cannot buy or sell bonds, so that the only asset in its possession at each point in time is capital. Thus, the budget constraint of household j is

$$c_t^j + i_t^j \leq (1 - \tau)(\underbrace{r_t k_t^j + w_t l_t^j + \pi_t^j}_{\equiv y_t^j}) + T_t^j,$$

where c_t^j denotes consumption, i_t^j investment, r_t the interest rate, k_t^j capital, w_t the wage rate, l_t^j the labor supply of household j, and π_t^j the dividends to which household j is entitled to. Note that the budget constraint states that the household can consume and invest up to the point in which it exhausts all its *disposable income*, that is, its total income minus the taxes. The capital stock accumulates according to

$$k_{t+1}^j = i_t^j + (1-\delta)k_t^j,$$

where $\delta \in (0,1)$. Each household j is endowed with an initial level of capital $k_0^j > 0$ and a unit of labor per period, that is, $l_t^j \leq 1$ for every t. Household j's lifetime utility is given by

$$\mathscr{U}_0^j = \sum_{t=0}^{+\infty} \beta^t U(c_t^j),$$

where $\beta \in (0,1)$ is the discount factor and U is Neoclassical. Moreover, the per-period utility is CRRA, that is,

$$U(c_t) \equiv \frac{c_t^{1-\theta}}{1-\theta},$$

for every t, where $\theta > 0$.

Firms are perfectly competitive and maximize profits. There is a continuum of mass M_t of firms. Each firm $m \in [0, M_t]$ has access to the same production technology, that is,

$$Y_t^m = F(K_t^m, L_t^m) = (K_t^m)^\alpha (L_t^m)^{1-\alpha},$$

where Y_t^m denotes firm-specific production, K_t^m capital, and L_t^m labor. Each firm observes the interest rate and the wage rate, and decides how much capital to rent and how much labor to employ, produces Y_t^m using the production technology, realizes profits Π_t^m and distributes them in the form of dividends. We assume that $\alpha \in (0,1)$ so that F is Neoclassical.

Markets clear according to the usual conditions, that is,

$$\int_0^1 k_t^j dj = \int_0^{M_t} K_t^m dm$$

for the capital market,

$$\int_0^1 l_t^j dj = \int_0^{M_t} L_t^m dm$$

for the labor market, and

$$\int_0^1 \pi_t^j dj = \int_0^{M_t} \Pi_t^m dm$$

for the profits and the dividends.

1. Suppose that there is no government. State the problem of the central planner in the centralized economy. Write down the Euler condition, the binding resource constraint, the transversality condition, and the initial condition. [Skip the natural nonnegativity constraints in the formulation of the problem of the central planner. Do not lose time in deriving the conditions.]

2. Let us go back to the decentralized competitive economy with government. Define the problem of household j, that is, list all the constraints it is subject to (natural nonnegativity constraints included), identify the objective function, and list the control variables. Define also the problem of firm m. Provide a definition for a general competitive equilibrium in this economy. [Describe thoroughly which variables are chosen by which agents and which others are taken as given. Remember to include the government.]

3. Simplify household j's problem considering the decision about the labor supply. Combine the constraints in order to simplify even further the feasible set of the household and derive the Karush-Kuhn-Tucker conditions for the simplified problem of household j. Elaborate these conditions in order to express explicitly the Euler condition, the transversality condition, the binding budget constraint, and the initial condition. [Do not limit yourself to the general case. Express these conditions in a parametrized functional form.]

4. Suppose from now on that the initial endowment of capital across households is the same and it is strictly positive, that is, $k_0^j = k_0 > 0$ for every $j \in [0, 1]$. What can we say about the differences in the optimal choices of consumption and capital at each point in time? Can we talk about a representative household in this economy? Given our assumptions on the budget constraint of the household at each point in time, would we be able to reformulate the equilibrium as an Arrow-Debreu equilibrium? [This is definitely a bonus question.]

5. Focus on the Euler condition for household j, which describes the optimal consumption behavior of households. How does consumption growth depend on the tax rate?

6. Compare the Euler condition for the generic household in the decentralized economy to the Euler condition in the centralized economy with no government. What should the tax rate be in order for the decentralized allocations to coincide with the allocations of the central planner? Give an intuition of your conclusion.

7. Derive the system of two difference equations that describes the dynamics of consumption and capital. Write down explicitly the bondary conditions for this system to admit a unique solution. Do the dynamics of consumption and capital depend on the distribution of the lump-sum transfers across households? [Do not report the general case, use the assumptions on production and preferences. You should obtain a system of two difference equations that depends on the endogenous variables c_t, c_{t+1}, k_t, k_{t+1}, and the exogenous parameters α, θ, δ, β, and τ.]

8. Consider the dynamics of two economies, one with τ_H and the other with τ_L, with $\tau_H > \tau_L$. Compare the steady state of the two economies. Is consumption at steady state higher with τ_L than with τ_H? Give an intuition for this conclusion.

9. Consider now the phase diagram for the dynamics of c_t and k_t. Suppose that the economy is in the steady state with τ_L. At some point $t = t_0$, the government decides to increase capital taxes up to τ_H and to keep them at the new level forever. How does consumption react in the transition from the old steady state to the new steady state?

10. Consider the long run dynamics, that is, the balanced growth path. Does tax policy, that is, the choice of the tax rate τ, affect the growth rate of the aggregate variables in the long run?