Universitat de Barcelona - Department of Economic Theory
Master in Economics

Macroeconomics I - Fall Semester 2011

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Problem Set 1. The Solow Model.

Question 1. The Solow model with Harrod-neutral technological progress.

Consider the Solow model that was presented in class. Allow for labor-augmenting technological progress so that

$$Y_t = F(K_t, A_t L_t) = K_t^{\alpha} \left(A_t L_t\right)^{1-\alpha}.$$

Labor grows at the constant rate n, that is,

$$L_{t+1} = (1+n)L_t,$$

for every t, with L_0 given. Moreover, technology grows at the constant rate g, that is,

$$A_{t+1} = (1+g)A_t,$$

for every t, with A_0 given. On the other hand, capital evolves over time according to the usual law of motion,

$$K_{t+1} = I_t + (1-\delta)K_t,$$

for every t, with K_0 taken as given.

1. Define and characterize a feasible allocation in this economy.

2. Suppose that the planner wants to implement the usual policy of the Solow model, that is, that the output at each period is divided in constant shares between consumption and investment. In other words,

$$I_t = sY_t$$

and

$$C_t = (1-s)Y_t,$$

with $s \in (0,1)$. Define and characterize an "optimal" allocation in this economy.

3. Suppose now to apply the normalization

$$k_t \equiv \frac{K_t}{A_t L_t},$$

where $A_t L_t$ is called the amount of *efficiency units of labor* and k_t is the *capital per efficiency unit of labor*. Characterize the policy rules of the "optimal" allocation according to the normalization. [You can use the approximation around the steady state ($k_{t+1} \approx k_t$) to express the policy rule for capital.]

4. Define and characterize the steady state(s) of this economy. Discuss existence and uniqueness of the steady state(s). [You can use graphical representations of the policy rule for capital or of the growth rate of the capital per efficiency unit of labor. Pay particular attention to the definition of k_t.]

5. Analyze the transitional dynamics of this economy. Discuss local and global stability of the steady state(s).

6. Analyze the balanced growth path of this economy, that is, derive the growth rate of capital K_t, output Y_t, consumption C_t, and investment I_t at the steady state.[You can approximate the growth rate of any variable X_t by $log(X_{t+1}) - log(X_t)$. Moreover, recall that $log(1+x) \approx x$ when x is "small."]

7. What is the growth rate of output per worker at the steady state? [Recall that output per worker is the ratio Y_t/L_t.]

Question 2. Factor payments in the Solow model with Harrod-neutral technological progress (Romer 1.9).

Suppose again that technology is labor-augmenting, that is,

$$Y = F(K, AL).$$

Assume that both labor and capital are paid their marginal products. Let w denotes $\partial F(K, AL)/\partial L$ and r denotes $\partial F(K, AL)/\partial K$.

1. Show that the marginal product of labor, w, is $A[f(k) - kf'(k)]$. [Define k wisely.]

2. Show that if both capital and labor are paid their marginal products, constant returns to scale imply that the total amount paid to the factors of production equals total output. That is, show that under constant returns, $wL + rK = F(K, AL)$.

3. The return r to capital is roughly constant over time, as are the shares of output going to capital and to labor. Does a Solow economy with labor-augmenting technological progress on a balanced growth path exhibit these properties? What are the growth rates of w and r on a balanced growth path?

4. Suppose the economy begins with a level of k less than k^*. As k moves toward k^*, is w growing at a rate greater than, less than, or equal to its growth rate on the balanced growth path? What about r?