Universitat de Barcelona - Department of Economic Theory
Master in Economics

Macroeconomics I - Fall Semester 2011

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Consider the neoclassical growth model. Assume that there is no population growth and normalize the size of the population to 1. Suppose that there is no technological growth either. We introduce a government in the model. The government taxes the households and distributes the fiscal revenues through lump-sum transfers. The taxes are collected through proportional taxation, that is, a constant fraction (the tax rate) of the object of taxation goes into taxes. There are three types of taxation, that is, capital taxes τ^K, labor taxes τ_L, and consumption taxes τ^C. Note that the tax rates are constant over time. The total fiscal revenue will then be $\tau^K r_t K_t + \tau^L w_t L_t + \tau^C C_t$. We suppose that the government runs a balanced budget policy, so that

$$\tau^K r_t K_t + \tau^L w_t L_t + \tau^C C_t = T_t, \tag{1}$$

where $T_t \equiv \int_0^1 T_t^j dj$ is the total amount of lump-sum transfers distributed to the mass of households.
Suppose that household j cannot buy or sell bonds, so that the only asset in its possession at each point in time is capital. Thus, the budget constraint of household j is

$$c_t^j + i_t^j \leq r_t k_t^j + w_t l_t^j + \pi_t^j - \tau^K r_t k_t^j - \tau^L w_t l_t^j - \tau^C c_t^j + T_t^j, \tag{2}$$

where c_t^j denotes consumption, i_t^j investment, k_t^j capital, r_t the interest rate, w_t the wage rate, l_t^j the labor supply of household j, and π_t^j the dividens to which household j is entitled to. Note that the budget constraint states that the household can consume and invest up to the point in which it exhausts all its disposable income, that is, its total income minus the taxes. The capital stock accumulates according to

$$k_{t+1}^j = i_t^j + (1-\delta)k_t^j, \tag{3}$$

where $\delta \in (0,1)$. Each household j is endowed with the same initial level of capital $k_0^j = k_0 > 0$ and a unit of labor per period, that is, $l_t^j \leq 1$ for every t. Household j's lifetime utility is given by

$$\mathscr{U}_0^j = \sum_{t=0}^{+\infty} \beta^t U(c_t^j), \tag{4}$$

where $\beta \in (0,1)$ is the discount factor. Moreover, the per-period utility function U is CRRA, that is,

$$U(c_t) \equiv \frac{c_t^{1-\theta}}{1-\theta}, \tag{5}$$

for every t, where $\theta > 0$.
Firms are perfectly competitive and maximize profits. There is a continuum of mass M_t of firms. Each firm $m \in [0, M_t]$ has access to the same production technology, that is,

$$Y_t^m = F(K_t^m, L_t^m) = (K_t^m)^\alpha (L_t^m)^{1-\alpha}, \tag{6}$$

where Y_t^m denotes firm-specific production, K_t^m capital, and L_t^m labor, for every m. Each firm observes the interest rate and the wage rate, and decides how much capital to rent and how much labor to employ, produces Y_t^m using the production technology, realizes profits Π_t^m and distributes them in the form of dividends. We assume that $\alpha \in (0,1)$.
Markets clear according to the usual conditions, that is,

$$\int_0^1 k_t^j dj = \int_0^{M_t} K_t^m dm \tag{7}$$

for the capital market,

$$\int_0^1 l_t^j dj = \int_0^{M_t} L_t^m dm \tag{8}$$

for the labor market, and

$$\int_0^1 \pi_t^j dj = \int_0^{M_t} \Pi_t^m dm \tag{9}$$

for the profits and the dividends.

1. Suppose that there is no government. State the problem of the central planner in the centralized economy without government. Write down the Euler condition, the binding resource constraint, the transversality condition, and the initial condition. [Skip the natural nonnegativity constraints in the formulation of the problem of the central planner. Do not lose time in deriving the conditions. Express all variables in per-capita terms.]

2. Let us go back to the decentralized competitive economy with government. Define the problem of household j, that is, list all the constraints it is subject to, identify the objective function, and list the control variables. [Remember to include all the natural nonnegativity constraints.]

3. Define the problem of firm m. Derive the First Order Conditions (FOCs) for the problem of firm m. Do the optimal choices of labor and capital depend on any tax rate? What can we say about the relationship between capital and labor across firms? What are the profits of firm m under perfect competition? [Do not limit yourself to the general case, use (5) and (6).]

4. Provide a definition for a general competitive equilibrium in this economy. [Describe thoroughly which variables are chosen by which agents and which others are taken as given. Remember to include the government, that is, (1).]

5. Analyze the decision about the labor supply of household j. Does the labor supply depend on the tax rate on labor, τ^L? How can we simplify household j's problem? [Try also to combine the constraints in order to simplify even further the feasible set of the household.]

6. Derive the Karush-Kuhn-Tucker conditions for the simplified problem of household j. Elaborate these conditions in order to express explicitly the Euler condition, the transversality condition, the binding budget constraint, and the initial condition. [Do not limit yourself to the general case. Apply the assumptions on the utility function and on the production function to express these conditions in a parametrized functional form.]

7. Given that the initial endowment of capital is the same across households, what can we say about the differences across households in the optimal choices of consumption and capital at each point in time? Can we say that we can drop the index j for household-specific consumption and capital? What would be the relation between household-specific variables and aggregate variables?

8. Focus on the Euler condition for household j, which describes the optimal consumption behavior of households. How does consumption growth depend on the different tax rates? Is the optimal intertemporal allocation of consumption affected by the tax rate on consumption? Suppose the tax rate τ^C had not been constant over time, that is, at each period t each household would face a different τ_t^C. Would your answer change about the intertemporal allocation of consumption change?

9. Compare the Euler condition for the generic household in the decentralized economy to the Euler condition in the centralized economy with no government. What should the tax rates be in order for the decentralized allocations to coincide with the allocations of the central planner? Can you think of a combination of τ^C and τ^K such that $\tau^C > 0$, $\tau^K > 0$, and yet the Euler condition for the decentralized economy coincides with the Euler condition of the centralized economy?

10. Focus on the FOCs of firm m. Do r_t and w_t depend on the tax rates in equilibrium? How do r_t and w_t change as capital per unit of labor k_t increases?

11. Derive the system of two difference equations that describes the dynamics of consumption and capital. Write down explicitly the boundary conditions for this system to admit a unique solution. Do the dynamics of consumption and capital depend on the distribution of the lump-sum transfers across households? [Do not report the general case, use the assumptions on production and preferences. You should obtain a system of two difference equations that depends on the endogenous variables c_t, c_{t+1}, k_t, k_{t+1}, and the exogenous parameters α, θ, δ, β, τ^C, τ^K, and τ^L.]

12. Consider the dynamics of two economies, one with a high capital tax rate τ_H^K and the other with a low capital tax rate τ_L^K, with $\tau_H^K > \tau_L^K$. Compare the steady state of the two economies. Is consumption at steady state higher with τ_L^K than with τ_H^K? Give an intuition for this conclusion. Do the same comparison with two different tax rates on consumption, that is, with τ_H^C and τ_L^C where $\tau_H^C > \tau_L^C$.

13. Consider now the phase diagram for the dynamics of c_t and k_t. Suppose that the economy is in the steady state with τ_L^K. At some point $t = t_0$, the government decides to increase capital taxes up to τ_H^K and to keep them at the new level forever. How does consumption react in the transition from the old steady state to the new steady state?

14. Consider the long run dynamics, that is, the balanced growth path. Does tax policy, that is, the choice of the tax rates τ^K, τ^C, and τ^L affect the growth rate of the aggregate variables in the long run?