Universitat de Barcelona - Department of Economic Theory
Master in Economics

# Macroeconomics I - Fall Semester 2011

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## FINAL EXAM - 14/12/2011

**Guidelines.** This exam is composed of two parts. In Part I, you have to choose one out of two problems and answer to all the related questions. Only the problem that you choose will be corrected. In Part II, you have to solve an exercise. Part I accounts for 100 out of 100 points, that is, you can reach full grade by simply solving only Part I. Part II can add up to 20 bonus points. You have 2 hours to complete the exam.

## PART I

### Problem 1.

Consider the Ramsey-Cass-Koopmans model in discrete time. Assume there is no population growth and no technological growth. A government taxes the households and distributes the fiscal revenues through lump-sum transfers. There are three types of taxation, that is, capital taxes $\tau^K$, labor taxes $\tau^L$, and consumption taxes $\tau^C$. Note that the tax rates are constant over time. The government's balanced budget is

$$\tau^K r_t K_t + \tau^L w_t L_t + \tau^C C_t = T_t, \tag{1}$$

where $T_t \equiv \int_0^1 T_t^j dj$ is the total amount of lump-sum transfers distributed to the mass 1 of households. We suppose that the transfer $T_t^j$ devoted to household $j$ is the same across households, that is, $T_t = \int_0^1 T_t dj$.
The budget constraint of household $j$ is

$$(1+\tau^C)c_t^j + i_t^j \leq (1-\tau^K)r_t k_t^j + (1-\tau^L)w_t l_t^j + \pi_t^j + T_t, \tag{2}$$

and the capital stock accumulates according to

$$k_{t+1}^j = i_t^j + (1-\delta)k_t^j, \tag{3}$$

where $\delta \in (0,1)$. Each household $j$ is endowed with the same initial level of capital $k_0^j = k_0 > 0$ and a unit of labor per period, that is, $l_t^j \leq 1$ for every $t$. Household $j$'s lifetime utility is given by

$$\mathscr{U}_0^j = \sum_{t=0}^{+\infty} \beta^t U(c_t^j, 1-l_t^j), \tag{4}$$

where $\beta \in (0,1)$ is the discount factor. Moreover, the per-period utility function $U$ is additively separable in consumption and leisure. In particular,

$$U(c_t^j, 1-l_t^j) \equiv \frac{(c_t^j)^{1-\theta}}{1-\theta} + \gamma(1-l_t^j), \tag{5}$$

for every $t$, where $\theta > 0$ and $\gamma > 0$.
Firms are perfectly competitive and maximize profits. There is a continuum of mass $M_t$ of firms. Each firm $m \in [0, M_t]$ has access to the same production technology, that is,

$$Y_t^m = F(K_t^m, L_t^m) = (K_t^m)^\alpha (L_t^m)^{1-\alpha}, \tag{6}$$

where $\alpha \in (0,1)$.
The market clearing conditions are

$$\int_0^1 k_t^j dj = \int_0^{M_t} K_t^m dm \quad \text{and} \quad \int_0^1 l_t^j dj = \int_0^{M_t} L_t^m dm \tag{7}$$

for capital and labor markets. Moreover,

$$\int_0^1 \pi_t^j dj = \int_0^{M_t} \Pi_t^m dm, \tag{8}$$

that is, profits equal dividends.

1. Suppose that we are in the centralized economy with no government. Write down the Euler condition, the condition for the labor supply, the binding resource constraint, the transversality condition, and the initial condition. [Do not lose time in stating the problem and deriving the conditions. Express all variables in per-capita terms.]

2. In the decentralized competitive economy with government, define the problem of household $j$, that is, list all the constraints it is subject to, identify the objective function, and list the control variables. [Remember to include all the natural nonnegativity constraints and also that leisure now enters the utility function.]

3. Define the problem of firm $m$. Derive the First Order Conditions (FOCs) for the problem of firm $m$. Do the optimal choices of labor and capital depend on any tax rate? What can we say about the relationship between capital and labor across firms? What are the profits of firm $m$ under perfect competition? Express the two prices as functions of capital per capita and labor per capita. [Do not limit yourself to the general case, use (5) and (6).]

4. Derive the FOCs for the problem of household $j$. Elaborate these conditions in order to express explicitly the Euler condition, the condition for the labor supply, the binding budget constraint, the transversality condition, and the initial condition. [Do not limit yourself to the general case. Apply the assumptions on the utility function to express these conditions in a parametrized functional form.]

5. Given that the initial endowment of capital and the transfers at each point in time are the same across households, can we drop the index $j$ for household-specific consumption, labor, and capital at equilibrium? Explain.

6. Substitute in the Euler condition of the representative household for the equilibrium prices. Is the optimal intertemporal allocation of consumption affected by the tax rate on consumption? What about the tax rates on capital and labor?

7. Focus on the condition for the labor supply of the representative household after substituting for the equilibrium prices. How does the labor supply depend on the tax rates?

8. Compare the Euler condition and the condition for the labor supply of the representative household in the decentralized economy to the same conditions in the centralized economy with no government. What should the tax rates be in order for the equilibrium allocations to coincide with the optimal allocations?

**Problem 2.**

Consider an endogenous growth economy in discrete time. There is no population growth and no exogenous technological change. There is perfect competition across firms and perfect depreciation of capital ($\delta = 1$). The source of growth is productive services provided by the government. The government may impose two proportional taxes on both capital and labor. On the one hand, we denote with $\tau^L$ the tax rate on labor and with $\tau^K$ the tax rate on capital. The government balanced budget writes

$$\tau^K r_t k_t + \tau^L w_t = g_t, \tag{9}$$

where $k_t \equiv Kt/Lt$ is the capital per capita, $r_t$ is the rental rate of capital, $w_t$ the wage per unit of labor, and $g_t$ is the (productive) government spending per capita. The resource constraint of the economy is

$$c_t + i_t + g_t \leq y_t. \tag{10}$$

The preferences of the representative household are represented by the utility function,

$$\mathscr{U}_0 = \sum_{t=0}^{+\infty} \beta^t U(c_t), \tag{11}$$

where $\beta \in (0, 1)$ is the discount factor and the per-period utility function $U$ is logarithmic,

$$U(c) = \log c.$$

The representative household faces a budget constraint,

$$c_t + k_{t+1} \leq (1 - \tau^K) r_t k_t + (1 - \tau^L) w_t, \tag{12}$$

where $i_t = k_{t+1}$ for every $t$ due to the perfect depreciation of capital.
The representative firm maximizes profits,

$$\Pi_t = Y_t - r_t K_t - w_t L_t, \tag{13}$$

given the production technology,

$$Y_t = A_t K_t^\alpha L_t^{1-\alpha}, \tag{14}$$

where $\alpha \in (0, 1)$ denotes the income share of capital and $A_t$ denotes aggregate productivity in period $t$.
We assume that the productivity of a firm increases with the level of public services, that is,

$$A_t = g_t^{1-\alpha}. \tag{15}$$

Note that the representative firm and the representative household take $A_t$ as exogenous, but in general equilibrium $A_t$ increases with $g_t$.

1. Characterize $r_t$ and $w_t$ from the First Order Conditions (FOCs) of the firm's problem. Combine them with (14) and (9) to express $g_t$, $y_t$, $r_t$, and $w_t$ exclusively in terms of $k_t$ and $(\tau^K, \tau^L)$.

2. Similarly, rewrite the resource constraint (10) in terms of $(c_t, k_t, k_{t+1})$ and $(\tau^K, \tau^L)$ alone.

3. Explain why this economy is an example of an AK-type economy. Explain how do $(\tau^K, \tau^L)$ affect $y_t$ and $r_t$.

4. Write the Euler condition that characterizes equilibrium consumption growth in terms of $(c_t, c_{t+1})$ and $(\tau^K, \tau^L)$ alone. How do $(\tau^K, \tau^L)$ affect the rate of consumption growth?

5. Combine your results from the previous points in order to express the long-run growth rate of the economy $\gamma = (c_{t+1} - c_t)/c_t = (y_{t+1} - y_t)/y_t$ and the consumption-output ratio $c/y = c_t/y_t$ as functions of $(\tau^K, \tau^L)$. [You should already have the long run growth rate of the economy from the previous point. To solve for $c/y$, use your resource constraint from the first question and divide through by $k_t$. You will then need to use the fact that on the balanced growth path, it must be that consumption and capital grow at the same rate in order to solve for $c/y$.]

6. Interpret the effects of $\tau^K$ and $\tau^L$ on $\gamma$ and $c/y$.

7. Fix $\tau^L = 0$ and let $\tau^K \in [0, 1]$. What value of $\tau^K$ maximizes $c/y$ and what maximizes $\gamma$? Next, fix $\tau^K = 0$ and let $\tau^L \in [0, 1]$. What value of $\tau^L$ maximizes $c/y$ and what maximizes $\gamma$? Why is there a difference?

8. Suppose that the government can choose freely both $\tau^K$ and $\tau^L$ so as to maximize social welfare. Discuss what are the trade-offs the government faces in choosing the optimal tax rates that maximize social welfare. Is it optimal to use both $\tau^L > 0$ and $\tau^K > 0$, or just one of them, and if only one which one?

## PART II

Consider the Stochastic Growth model we saw in class. Production is

$$Y_t = z_t F(K_t, L_t) = z_t K_t^{\alpha} L_t^{1-\alpha},$$

where $z_t$ is the realization in $t$ of a time-homogeneous Markov chain and $\alpha \in (0,1)$, for every $t$. The per-period utility function is

$$U(c_t) = \log c_t,$$

for every $t$. Moreover, there is perfect depreciation, that is, $\delta = 1$.

1. Derive the policy rules for the problem of the central planner, that is, express the optimal level of the control variables as a function of endogenous and exogenous states. Compute also the production in $t$ given the policy rules.

2. What do these equations tell us about the correlation between output, consumption, and investment? Explain.